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CALCULATION OF THE k-FACTORS FOR THE FAILURE PROBABILITIES

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ABSTRACT

Failure probabilities of various components are important parameters for the safety evaluation of a generating station. In practice, the failure probabilities of components are calculated from their failure rates, which can be identified only with a degree of uncertainty (they are random variables). As a consequence, the component failure probabilities are also random variables. The uncertainty of both the rates and the probabilities of failures are characterized by their medians and k-factors. While, in the Darlington Probabilistic Safety Evaluation project, the medians and k-factors of the estimated failure rates were known, the same for the failure probabilities were not given. However, this information is a required input to the computer code (SEP) which carries out the uncertainty analysis and therefore, the formulae to compute the medians and k-factors for the failure probabilities were derived. Also, since the SEP computer code assumes the failure probabilities to be log-normally distributed, this assumption needed to be validated. The log-normal assumption was tested via Monte Carlo simulations, and the results indicated that the assumption was dubious, at best. This may necessitate the selection of a more flexible computer code for future uncertainty analyses.

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1. INTRODUCTION

Failure probabilities of various components in a nuclear generating station are important parameters for the safety evaluation. These probabilities are, in general, derived from their corresponding failure rates. The failure rates, on the other hand, are usually estimated and therefore have a degree of uncertainty (they are random variables). Consequently, the failure probabilities are also random variables. In such a case, an uncertainty analysis is needed in a safety evaluation.

The medians and k-factors (the ratio of the 95 percentile and the median) characterize the uncertainties of both the component failure rates and the failure probabilities. Although the medians and k-factors of the failure rates are generally available, the same for the failure probabilities are not known immediately; they are, however, required inputs to the uncertainty analysis computer code **SEP** used in the Darlington Probabilistic Safety Evaluation project. In this report, the formulae to compute the medians and the k-factors of the failure probabilities are derived.

The computer code **SEP** assumes the failure probabilities to be log-normally distributed. This assumption is a subject for debate. Hence, this assumption is tested via the Monte Carlo simulation technique.

In summary, the objectives of this report were:

- to devise the formulae to calculate the medians and k-factors of the failure probabilities;
- to test the log-normal assumption for the failure probabilities.

2. CALCULATIONS OF THE FAILURE PROBABILITIES FROM FAILURE RATES

The failure probability of each component is derived from its corresponding failure rates. In this exercise, four types of failure probabilities were considered, namely, the failure probabilities of a dormant component, of a continuously running component, of a started dormant component and of an intermittently running component. Also, the probability of the occurrence of an unspecified event is also required in the uncertainty analysis.

These probabilities were computed via the following formulae which were provided by the Quality Engineering Department:

(A) Failure probability of a dormant component

$$p = 0.001\lambda_d(T_i/730 + T_r/8760) + T_t/24T_i$$

(B) Failure probability (or unavailability) of a continuously running component

$$p = \lambda_r T_r/(8.76\text{x}10^6 + \lambda_r T_r)$$

(C) Running failure probability of a dormant component (given successful start)

$$p = 1 - \exp\{-\lambda_r T_m/8.76\text{x}10^6\}$$

(D) Failure probability of an intermittently running component

$$p = Z[0.001\lambda_d(T_i/730 + T_r/8760) + T_t/24T_i] + X[\lambda_r T_r/(8.76\text{x}10^6 + \lambda_r T_r)]$$

(E) Probability of occurrence of an event

$$p = f_m T_e/8760$$

Where p is the failure probability or the unavailability as appropriate,
λ_d is the *dormant failure rate* (failures/1000 yrs),
λ_r is the *running failure rate* (failures/1000 yrs),
T_i is the *test interval* (days),
T_r is the *repair time* (hrs/failure),
T_t is the *test duration* (hrs/test),
T_m is the *mission time* (hrs),
Z is the *dormancy factor*,
$X = 1 - Z$ is the *operating factor*,
f_m is the *frequency of occurrence of the event*, (occ./yr),
T_e is the *average duration of event* (hrs/occ.).

3. FAILURE PROBABILITIES DEPENDING ON ONE FAILURE RATE

Let p denote the failure probability and λ the failure rate. If p is related to λ via a monotonic increasing[1] function f, ie $p = f(\lambda)$, then the k-factor of p can be determined by a simple substitution. In other words, suppose that λ_m and λ_k are respectively the given median and k-factor of λ. Let p_m and p_k denote the median and the k-factor of p. Note that $\lambda_k\lambda_m$ and $p_k p_m$ define the 95-percentile points for λ and p. Then,

$$\begin{aligned} 0.95 &= P\{\, p < p_k p_m \,\} \\ &= P\{\, f^{-1}(p) < f^{-1}(p_k p_m) \,\} \\ &= P\{\, \lambda < f^{-1}(p_k p_m) \,\} \end{aligned}$$

where f^{-1} is the inverse of f (since f is monotonic increasing, each value x corresponds to a unique $f(x)$ and hence its inverse exists). This implies that $f^{-1}(p_k p_m)$ is the upper 95-percentile point of λ, ie $\lambda_k\lambda_m = f^{-1}(p_k p_m)$. In other words, $p_k p_m = f(\lambda_k\lambda_m)$. Similarly, $p_m = f(\lambda_m)$. Hence, p_k, the k-factor for p is given by

$$p_k = f(\lambda_k\lambda_m)/f(\lambda_m)$$

Specifically, the following formulae can be easily established from the above:

---

1. A monotonic increasing function f possesses the property that $f(x) < f(y)$ if $x < y$.

(1) From equation (A) in Section 1, the formulae for p_m and p_k, the median and the k-factor of the failure probability of a dormant component respectively, are given by

$$p_m = 0.001\lambda_m(T_i/730 + T_r/8760) + T_t/24T_i$$

and $$p_k = [0.001\lambda_k\lambda_m(T_i/730 + T_r/8760) + T_t/24T_i]/p_m$$

where T_i, T_r and T_t are defined as in Section 1.

(2) From (B) in Section 1, the formulae for p_m and p_k, the median and the k-factor of the failure probability (or unavailability) of a continuously running component respectively, are given by

$$p_m = \lambda_m T_r/(8.76\text{x}10^6 + T_r\lambda_m)$$

and $$p_k = [\lambda_k\lambda_m T_r/(8.76\text{x}10^6 + \lambda_r\lambda_m T_r)]/p_m$$

where T_r is defined as in Section 1.

(3) From (C) in Section 1, the formulae for p_m and p_k, the median and the k-factor for the running failure probability of a dormant component (given successful start) respectively, are given by

$$p_m = 1 - \exp\{ -\lambda_m T_m/(8.76\text{x}10^6) \}$$

and $$p_k = [1 - \exp\{ -\lambda_k\lambda_m T_m/(8.76\text{x}10^6) \}]/p_m$$

where T_m is defined as in Section 1.

(4) From (E) in Section 1, the formulae for p_m and p_k, the median and the k-factor for the probability of an event respectively, are given by

$$p_m = \text{Median}(f_m)T_e/8760$$

p_k = the k-factor of the f_m

where T_e and f_m are defined in Section 1.

4. FAILURE PROBABILITY DEPENDING ON TWO FAILURE RATES

The failure probability of an intermittently running component (See (D) in Section 1) depends on the dormant failure rate and the running failure rate. In this case, the calculation of the k-factor is not as simple as the previous ones. The exact percentile point can be obtained from the following equation defined by a *Lebesgue-Stieltjes* integral:

$$\int_y^\infty F_1[ax - e - abXy/(c + by)]dF_2(y) = \alpha$$

where $\mathbf{F}_1$ and $\mathbf{F}_2$ are the cumulative distributions for λ_d and λ_r, respectively,

$a = 1000/[Z((T_i/730) + (T_r/8760))]$,

$b = T_r$,

$c = 8.76 \times 10^6$,

$e = aT_t/24T_m$,

and T_i, T_r, T_m, T_t, X and Z are defined in Section 1.

By solving the above equation for x with the values $\alpha = 0.5$ and $\alpha = 0.95$, the median and k-factor can be established.

To solve this equation, the functional forms of $\mathbf{F}_1$ and $\mathbf{F}_2$ must be specified. Since the occurrence of failures is assumed to be a Poisson process, the distribution functions $\mathbf{F}_1$ and $\mathbf{F}_2$ can be assumed to be χ^2. In this case, the solution of the above integral cannot be written in an algebraic form. Hence, the equation can only be solved numerically. This exercise would involve a great deal of time and effort, and was not allowable within the constraints of this study.

The **SEP** computer code for the uncertainty analysis assumes the failure probability to be log-normally distributed. This assumption allows the use of a simple algorithm instead of a general solution. Thus, using the properties of the log-normal distribution, and making the further assumption that the mechanisms which cause the dormant failure and the running failure are independent, the median and k-factor for the intermittent failure probability can be computed by the following steps:

1. Compute $m = p_m$ of (1) in Section 2
2. Compute $k = p_k$ of (1) in Section 2.
3. Compute $M = p_m$ of (2) in Section 2.
4. Compute $K = p_k$ of (2) of Section 2.
5. Define Z = the dormancy factor
6. Compute X = the operating factor = $1 - Z$
7. Compute $\delta = \ln(k)/1.65$
8. Compute $\Delta = \ln(K)/1.65$
9. Compute $e = m \exp\{ \delta^2/2 \}$
10. Compute $E = M \exp\{ \Delta^2/2 \}$
11. Compute $\theta = \exp\{ \delta^2 \}$

12. Compute $\Theta = \exp\{ \Delta^2 \}$

13. Compute $s = m[\theta(\theta - 1)]^{1/2}$

14. Compute $S = M[\Theta(\Theta - 1)]^{1/2}$

15. Compute $\mu = Ze + XE$

16. Compute $\eta = [Z^2 s^2 + X^2 S^2]^{1/2}$

17. Compute $\sigma = [\ln(1 + (\eta/\mu)^2)]^{1/2}$

Then the median and k-factor, denoted by p_m and p_k respectively, of the intermittent failure probability are given by the formulae:

$$p_m = \mu \exp\{ -\sigma^2/2 \}$$

$$p_k = \exp\{ 1.65\sigma \}$$

Remark The log-normal assumption was proven questionable in the next section. Due to the constraint of the available computer code, however, this assumption is necessary. Therefore, the above formulae can only be used with the understanding that errors may exist in the final outcomes of the uncertainty analysis.

5. TEST OF THE LOG-NORMAL ASSUMPTION

It was stated that failure probabilities were assumed to be log-normally distributed in the SEP computer code. But this assumption was not substantiated. Since the failure rate can be assumed to have a Chi-squared (χ^2) distribution, to test the log-normal assumption is to test whether the failure probability obtained from a χ^2 failure rate, via the transformations of (A) to (E) in Section 1, is log-normally distributed. To carry out this exercise, the Monte Carlo simulation technique was employed.

To perform the simulations, a random sample of 50 values were chosen from a χ^2 distribution, using three values for the degree of freedom: 5, 50, and 3000. Each simulation consisted of 100 iterations, where each iteration involved:

1. The selection of the sample of 50 values from a χ^2 distribution;
2. Calculating the failure probabilities using the formulas specified in Section 1
3. Obtaining the log of these failure probabilities;
4. Calculating the mean and standard deviation of a randomly-chosen half sample of these log values;
5. Obtaining the Kolmogorov-Smirnov-Durbin statistic for the standardized variable.

This procedure was performed on a MSDOS micro-computer using the SAS/PC software. The listing of the SAS program is given in the Appendix. The results of the simulations for each of

the formulae, which are given in Tables 1 and 2, indicate that most of the *rejection probabilities*[2] are quite high and thus the log-normal hypothesis is cast in doubt. The rejection criteria may be stated as follows: if the probability is greater than 95%, then the log-normal hypothesis is rejected; if the probability is between 80%-95%, then there is reasonable doubt that the log-normal assumption is true; if the probability is less than 80%, then there is insufficient evidence to reject the log-normal assumption.

Table 1

REJECTION PROBABILITIES FROM THE MONTE CARLO SIMULATIONS FOR EQUATIONS (A), (B) AND (C)

Equation\D.F.	5	50	3000
(A)	99%	50%	99%
(B)	95%	60%	96%
(C)	99%	99%	94%

Table 2

REJECTION PROBABILITIES FROM THE MONTE CARLO SIMULATIONS FOR THE EQUATION (D)

DF1\DF2	5	50	3000
5	99%	84%	94%
50	10%	19%	95%
3000	99%	99%	99%

Remark In equation (D), the failure probability of an intermittently running component is calculated from two failure rates, namely the running and the dormant failure rates. In Table 2, DF1 is the degree of freedom for the dormant failure rate and DF2 the running failure rate.

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2. In statistical terminology, this probability is called the confidence level of rejecting the log-normal hypothesis. In other words, a confidence level α implies that the chance of the log-normal hypothesis being true is $1 - \alpha$.

6. CONCLUSIONS

- Formulae were derived, and are given in Section 2, to compute the medians and the k-factors for the failure probabilities. They do not depend on the distributions of λ or p.
- The validity of the assumption that the failure probabilities are log-normally distributed was shown to be doubtful.
- The formulae given in Section 3, and based on the **SEP** code, are valid only if the distribution of the probabilities are approximately log-normal. Since there is no immediately available computer code that will accept arbitrary distributions, and modification of the **SEP** computer code is too time consuming, the log-normal assumption is indispensable at present, regardless of the above result.
- Another condition for the validity of the formulae in Section 3 is that the mechanisms for the dormant and running failures must be independent.
- For a reliable uncertainty analysis, a computer code flexible enough to accept arbitrary distributions for the failure probabilities needs to be developed.

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APPENDIX

SOURCE CODE FOR MONTE CARLO SIMULATION

```
FILENAME STDFILE 'D:\RSUE\STANDARD.DAT';
FILENAME SYSOUT  'D:\RSUE\SYSOUT.PRN';
*-------------------------------------------------------------------;
*  FORMULA (A); DEGREE OF FREEDOM = 5                               ;
*-------------------------------------------------------------------;
DATA SAMPLE;
  DEGREE=5;
  ITERATE=100;
  SMPLSIZE=50;
  ARRAY X(50) X1-X50;
  ARRAY LMDA(50) LMDA1-LMDA50;
  TI=1;
  TR=6;
  TT=1;
  A=0.001*((TI/730)+(TR/8760));
  B=TT/(24*TI);
  DO I=1 TO ITERATE;
    DO J=1 TO SMPLSIZE;
      LMDA(J)=CINV(RANUNI(0),DEGREE);
      P=(A*LMDA(J))+B;
      X(J)=LOG(P-B);
    END;
    M=MEAN(OF X1-X25);
    S=STD(OF X1-X25);
    DO J=1 TO SMPLSIZE;
      Y=(X(J)-M)/S;
      PNORM=PROBNORM(Y);
      FILE STDFILE;
      PUT @1 I @5 LMDA(J) 10.5 @16 X(J) 15.10
          @32 Y 15.10 @48 PNORM 15.10;
    END;
  END;
RUN;
DATA STDDATA;
  INFILE STDFILE;
  INPUT I 1-3 LMDA 5-14 X 16-30 Y 32-46 PNORM 48-62;
RUN;
PROC SORT DATA=STDDATA;
  BY I Y;
RUN;
DATA MAXIMUM;
  SET STDDATA;
  BY I Y;
  SMPLSIZE=50;
  J+1;
  D=SQRT(SMPLSIZE)*MAX(ABS(PNORM-(J/(SMPLSIZE+1))),
```

```
                        ABS(PNORM-((J-1)/(SMPLSIZE+1))));
  IF J=SMPLSIZE THEN J=0;
RUN;
PROC MEANS DATA=MAXIMUM MAX;
  VAR D;
  BY I;
  OUTPUT OUT=STATS MAX=MA;
RUN;
DATA _NULL_;
  SET STATS;
  FILE SYSOUT;
  PUT @1 '(A,5    )       : D(MAX)= ' @25 MA;
RUN;
*;
*------------------- ----------------------------------------------------;
*  FORMULA (A); DEGREE OF FREEDOM = 50                                    ;
*-------------------------------------------------------------------------;
DATA SAMPLE;
  DEGREE=50;
  ITERATE=100;
  SMPLSIZE=50;
  ARRAY X(50) X1-X50;
  ARRAY LMDA(50) LMDA1-LMDA50;
  TI=1;
  TR=6;
  TT=1;
  A=0.001*((TI/730)+(TR/8760));
  B=TT/(24*TI);
  DO I=1 TO ITERATE;
    DO J=1 TO SMPLSIZE;
      LMDA(J)=CINV(RANUNI(0),DEGREE);
      P=(A*LMDA(J))+B;
      X(J)=LOG(P-B);
    END;
    M=MEAN(OF X1-X25);
    S=STD(OF X1-X25);
    DO J=1 TO SMPLSIZE;
      Y=(X(J)-M)/S;
      PNORM=PROBNORM(Y);
      FILE STDFILE;
      PUT @1 I @5 LMDA(J) 10.5 @16 X(J) 15.10
          @32 Y 15.10 @48 PNORM 15.10;
    END;
  END;
RUN;
DATA STDDATA;
  INFILE STDFILE;
  INPUT I 1-3 LMDA 5-14 X 16-30 Y 32-46 PNORM 48-62;
RUN;
PROC SORT DATA=STDDATA;
  BY I Y;
```

```
RUN;
DATA MAXIMUM;
  SET STDDATA;
  BY I Y;
  SMPLSIZE=50;
  J+1;
  D=SQRT(SMPLSIZE)*MAX(ABS(PNORM-(J/(SMPLSIZE+1))),
                       ABS(PNORM-((J-1)/(SMPLSIZE+1))));
  IF J=SMPLSIZE THEN J=0;
RUN;
PROC MEANS DATA=MAXIMUM MAX;
  VAR D;
  BY I;
  OUTPUT OUT=STATS MAX=MA;
RUN;
DATA _NULL_;
  SET STATS;
  FILE SYSOUT MOD;
  PUT @1 '(A,50  )       : D(MAX)= ' @25 MA;
RUN;
*;
*-------------------------------------------------------------------;
*  FORMULA (A); DEGREE OF FREEDOM = 3000                            ;
*-------------------------------------------------------------------;
DATA SAMPLE;
  DEGREE=3;
  ITERATE=100;
  SMPLSIZE=50;
  ARRAY X(50) X1-X50;
  ARRAY LMDA(50) LMDA1-LMDA50;
  TI=1;
  TR=6;
  TT=1;
  A=((TI/730)+(TR/8760));
  B=TT/(24*TI);
  DO I=1 TO ITERATE;
    DO J=1 TO SMPLSIZE;
      LMDA(J)=CINV(RANUNI(0),DEGREE);
      P=(A*LMDA(J))+B;
      X(J)=LOG(P-B);
    END;
    M=MEAN(OF X1-X25);
    S=STD(OF X1-X25);
    DO J=1 TO SMPLSIZE;
      Y=(X(J)-M)/S;
      PNORM=PROBNORM(Y);
      FILE STDFILE;
      PUT @1 I @5 LMDA(J) 10.5 @16 X(J) 15.10
          @32 Y 15.10 @48 PNORM 15.10;
    END;
  END;
```

```
RUN;
DATA STDDATA;
  INFILE STDFILE;
  INPUT I 1-3 LMDA 5-14 X 16-30 Y 32-46 PNORM 48-62;
RUN;
PROC SORT DATA=STDDATA;
  BY I Y;
RUN;
DATA MAXIMUM;
  SET STDDATA;
  BY I Y;
  SMPLSIZE=50;
  J+1;
  D=SQRT(SMPLSIZE)*MAX(ABS(PNORM-(J/(SMPLSIZE+1))),
                       ABS(PNORM-((J-1)/(SMPLSIZE+1))));
  IF J=SMPLSIZE THEN J=0;
RUN;
PROC MEANS DATA=MAXIMUM MAX;
  VAR D;
  BY I;
  OUTPUT OUT=STATS MAX=MA;
RUN;
DATA _NULL_;
  SET STATS;
  FILE SYSOUT MOD;
  PUT @1 '(A,3000)       : D(MAX)= ' @25 MA;
RUN;
*;
*-------------------------------------------------------------------;
*  FORMULA (B); DEGREE OF FREEDOM = 5                               ;
*-------------------------------------------------------------------;
DATA SAMPLE;
  DEGREE=5;
  ITERATE=100;
  SMPLSIZE=50;
  ARRAY X(50) X1-X50;
  ARRAY LMDA(50) LMDA1-LMDA50;
  TR=6;
  A=8.76E6/TR;
  DO I=1 TO ITERATE;
    DO J=1 TO SMPLSIZE;
      LMDA(J)=CINV(RANUNI(0),DEGREE);
      P=LMDA(J)/(A+LMDA(J));
      X(J)=LOG(P);
    END;
    M=MEAN(OF X1-X25);
    S=STD(OF X1-X25);
    DO J=1 TO SMPLSIZE;
      Y=(X(J)-M)/S;
      PNORM=PROBNORM(Y);
      FILE STDFILE;
```

```
      PUT @1 I @5 LMDA(J) 10.5 @16 X(J) 15.10
          @32 Y 15.10 @48 PNORM 15.10;
    END;
  END;
RUN;
DATA STDDATA;
  INFILE STDFILE;
  INPUT I 1-3 LMDA 5-14 X 16-30 Y 32-46 PNORM 48-62;
RUN;
PROC SORT DATA=STDDATA;
  BY I Y;
RUN;
DATA MAXIMUM;
  SET STDDATA;
  BY I Y;
  SMPLSIZE=50;
  J+1;
  D=SQRT(SMPLSIZE)*MAX(ABS(PNORM-(J/(SMPLSIZE+1))),
                       ABS(PNORM-((J-1)/(SMPLSIZE+1))));
  IF J=SMPLSIZE THEN J=0;
RUN;
PROC MEANS DATA=MAXIMUM MAX;
  VAR D;
  BY I;
  OUTPUT OUT=STATS MAX=MA;
RUN;
DATA _NULL_;
  SET STATS;
  FILE SYSOUT MOD;
  PUT @1 '(B,5    )       : D(MAX)= ' @25 MA;
RUN;
*;
*-------------------------------------------------------------------;
*  FORMULA (B); DEGREE OF FREEDOM = 50                              ;
*-------------------------------------------------------------------;
DATA SAMPLE;
  DEGREE=50;
  ITERATE=100;
  SMPLSIZE=50;
  ARRAY X(50) X1-X50;
  ARRAY LMDA(50) LMDA1-LMDA50;
  TR=6;
  A=8.76E6/TR;
  DO I=1 TO ITERATE;
    DO J=1 TO SMPLSIZE;
      LMDA(J)=CINV(RANUNI(0),DEGREE);
      P=LMDA(J)/(A+LMDA(J));
      X(J)=LOG(P);
    END;
    M=MEAN(OF X1-X25);
    S=STD(OF X1-X25);
```

```
    DO J=1 TO SMPLSIZE;
      Y=(X(J)-M)/S;
      PNORM=PROBNORM(Y);
      FILE STDFILE;
      PUT @1 I @5 LMDA(J) 10.5 @16 X(J) 15.10
          @32 Y 15.10 @48 PNORM 15.10;
    END;
  END;
RUN;
DATA STDDATA;
  INFILE STDFILE;
  INPUT I 1-3 LMDA 5-14 X 16-30 Y 32-46 PNORM 48-62;
RUN;
PROC SORT DATA=STDDATA;
  BY I Y;
RUN;
DATA MAXIMUM;
  SET STDDATA;
  BY I Y;
  SMPLSIZE=50;
  J+1;
  D=SQRT(SMPLSIZE)*MAX(ABS(PNORM-(J/(SMPLSIZE+1))),
                       ABS(PNORM-((J-1)/(SMPLSIZE+1))));
  IF J=SMPLSIZE THEN J=0;
RUN;
PROC MEANS DATA=MAXIMUM MAX;
  VAR D;
  BY I;
  OUTPUT OUT=STATS MAX=MA;
RUN;
DATA _NULL_;
  SET STATS;
  FILE SYSOUT MOD;
  PUT @1 '(8,50  )       : D(MAX)= ' @25 MA;
RUN;
*;
*-------------------------------------------------------------------;
*  FORMULA (8); DEGREE OF FREEDOM = 3000                            ;
*-------------------------------------------------------------------;
DATA SAMPLE;
  DEGREE=3;
  ITERATE=100;
  SMPLSIZE=50;
  ARRAY X(50) X1-X50;
  ARRAY LMDA(50) LMDA1-LMDA50;
  TR=6;
  A=8.76E3/TR;
  DO I=1 TO ITERATE;
    DO J=1 TO SMPLSIZE;
      LMDA(J)=CINV(RANUNI(0),DEGREE);
      P=LMDA(J)/(A+LMDA(J));
```

```
      X(J)=LOG(P);
    END;
    M=MEAN(OF X1-X25);
    S=STD(OF X1-X25);
    DO J=1 TO SMPLSIZE;
      Y=(X(J)-M)/S;
      PNORM=PROBNORM(Y);
      FILE STDFILE;
      PUT @1 I @5 LMDA(J) 10.5 @16 X(J) 15.10
          @32 Y 15.10 @48 PNORM 15.10;
    END;
  END;
RUN;
DATA STDDATA;
  INFILE STDFILE;
  INPUT I 1-3 LMDA 5-14 X 16-30 Y 32-46 PNORM 48-62;
RUN;
PROC SORT DATA=STDDATA;
  BY I Y;
RUN;
DATA MAXIMUM;
  SET STDDATA;
  BY I Y;
  SMPLSIZE=50;
  J+1;
  D=SQRT(SMPLSIZE)*MAX(ABS(PNORM-(J/(SMPLSIZE+1))),
                       ABS(PNORM-((J-1)/(SMPLSIZE+1))));
  IF J=SMPLSIZE THEN J=0;
RUN;
PROC MEANS DATA=MAXIMUM MAX;
  VAR D;
  BY I;
  OUTPUT OUT=STATS MAX=MA;
RUN;
DATA _NULL_;
  SET STATS;
  FILE SYSOUT MOD;
  PUT @1 '(B,3000)       : D(MAX)= ' @25 MA;
RUN;
*;
*-------------------------------------------------------------------------;
*  FORMULA (C); DEGREE OF FREEDOM = 5                                     ;
*-------------------------------------------------------------------------;
DATA SAMPLE;
  DEGREE=5;
  ITERATE=100;
  SMPLSIZE=50;
  ARRAY X(50) X1-X50;
  ARRAY LMDA(50) LMDA1-LMDA50;
  TR=6;
  A=TR/8.76E6;
```

```
 DO I=1 TO ITERATE;
   DO J=1 TO SMPLSIZE;
     LMDA(J)=CINV(RANUNI(0),DEGREE);
     P=1-EXP((-1)*A*LMDA(J));
     X(J)=LOG(P);
   END;
   M=MEAN(OF X1-X25);
   S=STD(OF X1-X25);
   DO J=1 TO SMPLSIZE;
     Y=(X(J)-M)/S;
     PNORM=PROBNORM(Y);
     FILE STDFILE;
     PUT @1 I @5 LMDA(J) 10.5 @16 X(J) 15.10
         @32 Y 15.10 @48 PNORM 15.10;
   END;
  END;
RUN;
DATA STDDATA;
  INFILE STDFILE;
  INPUT I 1-3 LMDA 5-14 X 16-30 Y 32-46 PNORM 48-62;
RUN;
PROC SORT DATA=STDDATA;
  BY I Y;
RUN;
DATA MAXIMUM;
  SET STDDATA;
  BY I Y;
  SMPLSIZE=50;
  J+1;
  D=SQRT(SMPLSIZE)*MAX(ABS(PNORM-(J/(SMPLSIZE+1))),
                       ABS(PNORM-((J-1)/(SMPLSIZE+1))));
  IF J=SMPLSIZE THEN J=0;
RUN;
PROC MEANS DATA=MAXIMUM MAX;
  VAR D;
  BY I;
  OUTPUT OUT=STATS MAX=MA;
RUN;
DATA _NULL_;
  SET STATS;
  FILE SYSOUT MOD;
  PUT @1 '(C,5    )       : D(MAX)= ' @25 MA;
RUN;
*;
*-------------------------------------------------------------------;
*  FORMULA (C); DEGREE OF FREEDOM = 50                              ;
*-------------------------------------------------------------------;
DATA SAMPLE;
  DEGREE=50;
  ITERATE=100;
  SMPLSIZE=50;
```

```
  ARRAY X(50) X1-X50;
  ARRAY LMDA(50) LMDA1-LMDA50;
  TR=6;
  A=TR/8.76E6;
  DO I=1 TO ITERATE;
    DO J=1 TO SMPLSIZE;
      LMDA(J)=CINV(RANUNI(0),DEGREE);
      P=1-EXP((-1)*A*LMDA(J));
      X(J)=LOG(P);
    END;
    M=MEAN(OF X1-X25);
    S=STD(OF X1-X25);
    DO J=1 TO SMPLSIZE;
      Y=(X(J)-M)/S;
      PNORM=PROBNORM(Y);
      FILE STDFILE;
      PUT @1 I @5 LMDA(J) 10.5 @16 X(J) 15.10
          @32 Y 15.10 @48 PNORM 15.10;
    END;
  END;
RUN;
DATA STDDATA;
  INFILE STDFILE;
  INPUT I 1-3 LMDA 5-14 X 16-30 Y 32-46 PNORM 48-62;
RUN;
PROC SORT DATA=STDDATA;
  BY I Y;
RUN;
DATA MAXIMUM;
  SET STDDATA;
  BY I Y;
  SMPLSIZE=50;
  J+1;
  D=SQRT(SMPLSIZE)*MAX(ABS(PNORM-(J/(SMPLSIZE+1))),
                       ABS(PNORM-((J-1)/(SMPLSIZE+1))));
  IF J=SMPLSIZE THEN J=0;
RUN;
PROC MEANS DATA=MAXIMUM MAX;
  VAR D;
  BY I;
  OUTPUT OUT=STATS MAX=MA;
RUN;
DATA _NULL_;
  SET STATS;
  FILE SYSOUT MOD;
  PUT @1 '(C,50  )       : D(MAX)= ' @25 MA;
RUN;
*;
*-------------------------------------------------------------------;
*  FORMULA (C); DEGREE OF FREEDOM = 3000                            ;
*-------------------------------------------------------------------;
```

```
DATA SAMPLE;
  DEGREE=3;
  ITERATE=100;
  SMPLSIZE=50;
  ARRAY X(50) X1-X50;
  ARRAY LMDA(50) LMDA1-LMDA50;
  TR=6;
  A=TR/8.76E3;
  DO I=1 TO ITERATE;
    DO J=1 TO SMPLSIZE;
      LMDA(J)=CINV(RANUNI(0),DEGREE);
      P=1-EXP((-1)*A*LMDA(J));
      X(J)=LOG(P);
    END;
    M=MEAN(OF X1-X25);
    S=STD(OF X1-X25);
    DO J=1 TO SMPLSIZE;
      Y=(X(J)-M)/S;
      PNORM=PROBNORM(Y);
      FILE STDFILE;
      PUT @1 I @5 LMDA(J) 10.5 @16 X(J) 15.10
          @32 Y 15.10 @48 PNORM 15.10;
    END;
  END;
RUN;
DATA STDDATA;
  INFILE STDFILE;
  INPUT I 1-3 LMDA 5-14 X 16-30 Y 32-46 PNORM 48-62;
RUN;
PROC SORT DATA=STDDATA;
  BY I Y;
RUN;
DATA MAXIMUM;
  SET STDDATA;
  BY I Y;
  SMPLSIZE=50;
  J+1;
  D=SQRT(SMPLSIZE)*MAX(ABS(PNORM-(J/(SMPLSIZE+1))),
                       ABS(PNORM-((J-1)/(SMPLSIZE+1))));
  IF J=SMPLSIZE THEN J=0;
RUN;
PROC MEANS DATA=MAXIMUM MAX;
  VAR D;
  BY I;
  OUTPUT OUT=STATS MAX=MA;
RUN;
DATA _NULL_;
  SET STATS;
  FILE SYSOUT MOD;
  PUT @1 '(C,3000)       : D(MAX)= ' @25 MA;
RUN;
```

```
*;
*-------------------------------------------------------------------------;
*  FORMULA (D); DEGREE OF FREEDOM (1) = 5                                 ;
*               DEGREE OF FREEDOM (2) = 5                                 ;
*-------------------------------------------------------------------------;
DATA SAMPLE;
  DEGREE1=5;
  DEGREE2=5;
  ITERATE=100;
  SMPLSIZE=50;
  ARRAY X(50) X1-X50;
  ARRAY LMDAD(50) LMDAD1-LMDAD50;
  ARRAY LMDAR(50) LMDAR1-LMDAR50;
  TI=1;
  TR=6;
  TT=1;
  A=0.001*((TI/730)+(TR/8760));
  B=TT/(24*TI);
  C=8.76E6/TR;
  Z=0.5;
  XX=1.0-Z;
  DO I=1 TO ITERATE;
    DO J=1 TO SMPLSIZE;
      LMDAD(J)=CINV(RANUNI(0),DEGREE1);
      LMDAR(J)=CINV(RANUNI(0),DEGREE2);
      P=Z*((A*LMDAD(J))+B)+XX*(LMDAR(J)/(C+LMDAR(J)));
      X(J)=LOG(P);
    END;
    M=MEAN(OF X1-X25);
    S=STD(OF X1-X25);
    DO J=1 TO SMPLSIZE;
      Y=(X(J)-M)/S;
      PNORM=PROBNORM(Y);
      FILE STDFILE;
      PUT @1 I @5 LMDA(J) 10.5 @16 X(J) 15.10
          @32 Y 15.10 @48 PNORM 15.10;
    END;
  END;
RUN;
DATA STDDATA;
  INFILE STDFILE;
  INPUT I 1-3 LMDA 5-14 X 16-30 Y 32-46 PNORM 48-62;
RUN;
PROC SORT DATA=STDDATA;
  BY I Y;
RUN;
DATA MAXIMUM;
  SET STDDATA;
  BY I Y;
  SMPLSIZE=50;
  J+1;
```

```
  D=SQRT(SMPLSIZE)*MAX(ABS(PNORM-(J/(SMPLSIZE+1))),
                       ABS(PNORM-((J-1)/(SMPLSIZE+1))));
  IF J=SMPLSIZE THEN J=0;
RUN;
PROC MEANS DATA=MAXIMUM MAX;
  VAR D;
  BY I;
  OUTPUT OUT=STATS MAX=MA;
RUN;
DATA _NULL_;
  SET STATS;
  FILE SYSOUT MOD;
  PUT @1 '(D,5    ,5    ) : D(MAX)= ' @25 MA;
RUN;
*;
*-------------------------------------------------------------------;
*  FORMULA (D); DEGREE OF FREEDOM (1) = 5                           ;
*               DEGREE OF FREEDOM (2) = 50                          ;
*-------------------------------------------------------------------;
DATA SAMPLE;
  DEGREE1=5;
  DEGREE2=50;
  ITERATE=100;
  SMPLSIZE=50;
  ARRAY X(50) X1-X50;
  ARRAY LMDAD(50) LMDAD1-LMDAD50;
  ARRAY LMDAR(50) LMDAR1-LMDAR50;
  TI=1;
  TR=6;
  TT=1;
  A=0.001*((TI/730)+(TR/8760));
  B=TT/(24*TI);
  C=8.76E6/TR;
  Z=0.5;
  XX=1.0-Z;
  DO I=1 TO ITERATE;
    DO J=1 TO SMPLSIZE;
      LMDAD(J)=CINV(RANUNI(0),DEGREE1);
      LMDAR(J)=CINV(RANUNI(0),DEGREE2);
      P=Z*((A*LMDAD(J))+B)+XX*(LMDAR(J)/(C+LMDAR(J)));
      X(J)=LOG(P);
    END;
    M=MEAN(OF X1-X25);
    S=STD(OF X1-X25);
    DO J=1 TO SMPLSIZE;
      Y=(X(J)-M)/S;
      PNORM=PROBNORM(Y);
      FILE STDFILE;
      PUT @1 I @5 LMDA(J) 10.5 @16 X(J) 15.10
          @32 Y 15.10 @48 PNORM 15.10;
    END;
```

```
  END;
RUN;
DATA STDDATA;
  INFILE STDFILE;
  INPUT I 1-3 LMDA 5-14 X 16-30 Y 32-46 PNORM 48-62;
RUN;
PROC SORT DATA=STDDATA;
  BY I Y;
RUN;
DATA MAXIMUM;
  SET STDDATA;
  BY I Y;
  SMPLSIZE=50;
  J+1;
  D=SQRT(SMPLSIZE)*MAX(ABS(PNORM-(J/(SMPLSIZE+1))),
                       ABS(PNORM-((J-1)/(SMPLSIZE+1))));
  IF J=SMPLSIZE THEN J=0;
RUN;
PROC MEANS DATA=MAXIMUM MAX;
  VAR D;
  BY I;
  OUTPUT OUT=STATS MAX=MA;
RUN;
DATA _NULL_;
  SET STATS;
  FILE SYSOUT MOD;
  PUT @1 '(D,5    ,50   ) : D(MAX)= ' @25 MA;
RUN;
*;
*-------------------------------------------------------------------;
*  FORMULA (D); DEGREE OF FREEDOM (1) = 5                           ;
*               DEGREE OF FREEDOM (2) = 3000                        ;
*-------------------------------------------------------------------;
DATA SAMPLE;
  DEGREE1=5;
  DEGREE2=3;
  ITERATE=100;
  SMPLSIZE=50;
  ARRAY X(50) X1-X50;
  ARRAY LMDAD(50) LMDAD1-LMDAD50;
  ARRAY LMDAR(50) LMDAR1-LMDAR50;
  TI=1;
  TR=6;
  TT=1;
  A=0.001*((TI/730)+(TR/8760));
  B=TT/(24*TI);
  C=8.76E3/TR;
  Z=0.5;
  XX=1.0-Z;
  DO I=1 TO ITERATE;
    DO J=1 TO SMPLSIZE;
```

```
      LMDAD(J)=CINV(RANUNI(0),DEGREE1);
      LMDAR(J)=CINV(RANUNI(0),DEGREE2);
      P=Z*((A*LMDAD(J))+B)+XX*(LMDAR(J)/(C+LMDAR(J)));
      X(J)=LOG(P);
    END;
    M=MEAN(OF X1-X25);
    S=STD(OF X1-X25);
    DO J=1 TO SMPLSIZE;
      Y=(X(J)-M)/S;
      PNORM=PROBNORM(Y);
      FILE STDFILE;
      PUT @1 I @5 LMDA(J) 10.5 @16 X(J) 15.10
          @32 Y 15.10 @48 PNORM 15.10;
    END;
  END;
RUN;
DATA STDDATA;
  INFILE STDFILE;
  INPUT I 1-3 LMDA 5-14 X 16-30 Y 32-46 PNORM 48-62;
RUN;
PROC SORT DATA=STDDATA;
  BY I Y;
RUN;
DATA MAXIMUM;
  SET STDDATA;
  BY I Y;
  SMPLSIZE=50;
  J+1;
  D=SQRT(SMPLSIZE)*MAX(ABS(PNORM-(J/(SMPLSIZE+1))),
                       ABS(PNORM-((J-1)/(SMPLSIZE+1))));
  IF J=SMPLSIZE THEN J=0;
RUN;
PROC MEANS DATA=MAXIMUM MAX;
  VAR D;
  BY I;
  OUTPUT OUT=STATS MAX=MA;
RUN;
DATA _NULL_;
  SET STATS;
  FILE SYSOUT MOD;
  PUT @1 '(D,5    ,3000) : D(MAX)= ' @25 MA;
RUN;
*;
*-------------------------------------------------------------------;
*  FORMULA (D); DEGREE OF FREEDOM (1) = 50                          ;
*               DEGREE OF FREEDOM (2) = 5                           ;
*-------------------------------------------------------------------;
DATA SAMPLE;
  DEGREE1=50;
  DEGREE2=5;
  ITERATE=100;
```

```
    SMPLSIZE=50;
    ARRAY X(50) X1-X50;
    ARRAY LMDAD(50) LMDAD1-LMDAD50;
    ARRAY LMDAR(50) LMDAR1-LMDAR50;
    TI=1;
    TR=6;
    TT=1;
    A=0.001*((TI/730)+(TR/8760));
    B=TT/(24*TI);
    C=8.76E6/TR;
    Z=0.5;
    XX=1.0-Z;
    DO I=1 TO ITERATE;
      DO J=1 TO SMPLSIZE;
        LMDAD(J)=CINV(RANUNI(0),DEGREE1);
        LMDAR(J)=CINV(RANUNI(0),DEGREE2);
        P=Z*((A*LMDAD(J))+B)+XX*(LMDAR(J)/(C+LMDAR(J)));
        X(J)=LOG(P);
      END;
      M=MEAN(OF X1-X25);
      S=STD(OF X1-X25);
      DO J=1 TO SMPLSIZE;
        Y=(X(J)-M)/S;
        PNORM=PROBNORM(Y);
        FILE STDFILE;
        PUT @1 I @5 LMDA(J) 10.5 @16 X(J) 15.10
            @32 Y 15.10 @48 PNORM 15.10;
      END;
    END;
  RUN;
  DATA STDDATA;
    INFILE STDFILE;
    INPUT I 1-3 LMDA 5-14 X 16-30 Y 32-46 PNORM 48-62;
  RUN;
  PROC SORT DATA=STDDATA;
    BY I Y;
  RUN;
  DATA MAXIMUM;
    SET STDDATA;
    BY I Y;
    SMPLSIZE=50;
    J+1;
    D=SQRT(SMPLSIZE)*MAX(ABS(PNORM-(J/(SMPLSIZE+1))),
                         ABS(PNORM-((J-1)/(SMPLSIZE+1))));
    IF J=SMPLSIZE THEN J=0;
  RUN;
  PROC MEANS DATA=MAXIMUM MAX;
    VAR D;
    BY I;
    OUTPUT OUT=STATS MAX=MA;
  RUN;
```

```
DATA _NULL_;
  SET STATS;
  FILE SYSOUT MOD;
  PUT @1 '(D,50   ,5    ) : D(MAX)= ' @25 MA;
RUN;
*;
*-------------------------------------------------------------------------;
*  FORMULA (D); DEGREE OF FREEDOM (1) = 50                                ;
*               DEGREE OF FREEDOM (2) = 50                                ;
*-------------------------------------------------------------------------;
DATA SAMPLE;
  DEGREE1=50;
  DEGREE2=50;
  ITERATE=100;
  SMPLSIZE=50;
  ARRAY X(50) X1-X50;
  ARRAY LMDAD(50) LMDAD1-LMDAD50;
  ARRAY LMDAR(50) LMDAR1-LMDAR50;
  TI=1;
  TR=6;
  TT=1;
  A=0.001*((TI/730)+(TR/8760));
  B=TT/(24*TI);
  C=8.76E6/TR;
  Z=0.5;
  XX=1.0-Z;
  DO I=1 TO ITERATE;
    DO J=1 TO SMPLSIZE;
      LMDAD(J)=CINV(RANUNI(0),DEGREE1);
      LMDAR(J)=CINV(RANUNI(0),DEGREE2);
      P=Z*((A*LMDAD(J))+B)+XX*(LMDAR(J)/(C+LMDAR(J)));
      X(J)=LOG(P);
    END;
    M=MEAN(OF X1-X25);
    S=STD(OF X1-X25);
    DO J=1 TO SMPLSIZE;
      Y=(X(J)-M)/S;
      PNORM=PROBNORM(Y);
      FILE STDFILE;
      PUT @1 I @5 LMDA(J) 10.5 @16 X(J) 15.10
          @32 Y 15.10 @48 PNORM 15.10;
    END;
  END;
RUN;
DATA STDDATA;
  INFILE STDFILE;
  INPUT I 1-3 LMDA 5-14 X 16-30 Y 32-46 PNORM 48-62;
RUN;
PROC SORT DATA=STDDATA;
  BY I Y;
RUN;
```

```
DATA MAXIMUM;
  SET STDDATA;
  BY I Y;
  SMPLSIZE=50;
  J+1;
  D=SQRT(SMPLSIZE)*MAX(ABS(PNORM-(J/(SMPLSIZE+1))),
                       ABS(PNORM-((J-1)/(SMPLSIZE+1))));
  IF J=SMPLSIZE THEN J=0;
RUN;
PROC MEANS DATA=MAXIMUM MAX;
  VAR D;
  BY I;
  OUTPUT OUT=STATS MAX=MA;
RUN;
DATA _NULL_;
  SET STATS;
  FILE SYSOUT MOD;
  PUT @1 '(D,50  ,50  ) : D(MAX)= ' @25 MA;
RUN;
*;
*-------------------------------------------------------------------;
*  FORMULA (D); DEGREE OF FREEDOM (1) = 50                          ;
*               DEGREE OF FREEDOM (2) = 3000                        ;
*-------------------------------------------------------------------;
DATA SAMPLE;
  DEGREE1=50;
  DEGREE2=3;
  ITERATE=100;
  SMPLSIZE=50;
  ARRAY X(50) X1-X50;
  ARRAY LMDAD(50) LMDAD1-LMDAD50;
  ARRAY LMDAR(50) LMDAR1-LMDAR50;
  TI=1;
  TR=6;
  TT=1;
  A=0.001*((TI/730)+(TR/8760));
  B=TT/(24*TI);
  C=8.76E3/TR;
  Z=0.5;
  XX=1.0-Z;
  DO I=1 TO ITERATE;
    DO J=1 TO SMPLSIZE;
      LMDAD(J)=CINV(RANUNI(0),DEGREE1);
      LMDAR(J)=CINV(RANUNI(0),DEGREE2);
      P=Z*((A*LMDAD(J))+B)+XX*(LMDAR(J)/(C+LMDAR(J)));
      X(J)=LOG(P);
    END;
    M=MEAN(OF X1-X25);
    S=STD(OF X1-X25);
    DO J=1 TO SMPLSIZE;
      Y=(X(J)-M)/S;
```

```
      PNORM=PROBNORM(Y);
      FILE STDFILE;
      PUT @1 I @5 LMDA(J) 10.5 @16 X(J) 15.10
          @32 Y 15.10 @48 PNORM 15.10;
    END;
  END;
RUN;
DATA STDDATA;
  INFILE STDFILE;
  INPUT I 1-3 LMDA 5-14 X 16-30 Y 32-46 PNORM 48-62;
RUN;
PROC SORT DATA=STDDATA;
  BY I Y;
RUN;
DATA MAXIMUM;
  SET STDDATA;
  BY I Y;
  SMPLSIZE=50;
  J+1;
  D=SQRT(SMPLSIZE)*MAX(ABS(PNORM-(J/(SMPLSIZE+1))),
                       ABS(PNORM-((J-1)/(SMPLSIZE+1))));
  IF J=SMPLSIZE THEN J=0;
RUN;
PROC MEANS DATA=MAXIMUM MAX;
  VAR D;
  BY I;
  OUTPUT OUT=STATS MAX=MA;
RUN;
DATA _NULL_;
  SET STATS;
  FILE SYSOUT MOD;
  PUT @1 '(D,50  ,3000) : D(MAX)= ' @25 MA;
RUN;
*;
*-------------------------------------------------------------------;
*  FORMULA (D); DEGREE OF FREEDOM (1) = 3000                        ;
*               DEGREE OF FREEDOM (2) = 5                           ;
*-------------------------------------------------------------------;
DATA SAMPLE;
  DEGREE1=3;
  DEGREE2=5;
  ITERATE=100;
  SMPLSIZE=50;
  ARRAY X(50) X1-X50;
  ARRAY LMDAD(50) LMDAD1-LMDAD50;
  ARRAY LMDAR(50) LMDAR1-LMDAR50;
  TI=1;
  TR=6;
  TT=1;
  A=((TI/730)+(TR/8760));
  B=TT/(24*TI);
```

```
  C=8.76E6/TR;
  Z=0.5;
  XX=1.0-Z;
  DO I=1 TO ITERATE;
    DO J=1 TO SMPLSIZE;
      LMDAD(J)=CINV(RANUNI(0),DEGREE1);
      LMDAR(J)=CINV(RANUNI(0),DEGREE2);
      P=Z*((A*LMDAD(J))+B)+XX*(LMDAR(J)/(C+LMDAR(J)));
      X(J)=LOG(P);
    END;
    M=MEAN(OF X1-X25);
    S=STD(OF X1-X25);
    DO J=1 TO SMPLSIZE;
      Y=(X(J)-M)/S;
      PNORM=PROBNORM(Y);
      FILE STDFILE;
      PUT @1 I @5 LMDA(J) 10.5 @16 X(J) 15.10
          @32 Y 15.10 @48 PNORM 15.10;
    END;
  END;
RUN;
DATA STDDATA;
  INFILE STDFILE;
  INPUT I 1-3 LMDA 5-14 X 16-30 Y 32-46 PNORM 48-62;
RUN;
PROC SORT DATA=STDDATA;
  BY I Y;
RUN;
DATA MAXIMUM;
  SET STDDATA;
  BY I Y;
  SMPLSIZE=50;
  J+1;
  D=SQRT(SMPLSIZE)*MAX(ABS(PNORM-(J/(SMPLSIZE+1))),
                       ABS(PNORM-((J-1)/(SMPLSIZE+1))));
  IF J=SMPLSIZE THEN J=0;
RUN;
PROC MEANS DATA=MAXIMUM MAX;
  VAR D;
  BY I;
  OUTPUT OUT=STATS MAX=MA;
RUN;
DATA _NULL_;
  SET STATS;
  FILE SYSOUT MOD;
  PUT @1 '(D,3000,5    ) : D(MAX)= ' @25 MA;
RUN;
*;
*-------------------------------------------------------------------;
*  FORMULA (D); DEGREE OF FREEDOM (1) = 3000                        ;
*               DEGREE OF FREEDOM (2) = 50                          ;
```

```
*-------------------------------------------------------------------;
DATA SAMPLE;
  DEGREE1=3;
  DEGREE2=50;
  ITERATE=100;
  SMPLSIZE=50;
  ARRAY X(50) X1-X50;
  ARRAY LMDAD(50) LMDAD1-LMDAD50;
  ARRAY LMDAR(50) LMDAR1-LMDAR50;
  TI=1;
  TR=6;
  TT=1;
  A=((TI/730)+(TR/8760));
  B=TT/(24*TI);
  C=8.76E6/TR;
  Z=0.5;
  XX=1.0-Z;
  DO I=1 TO ITERATE;
    DO J=1 TO SMPLSIZE;
      LMDAD(J)=CINV(RANUNI(0),DEGREE1);
      LMDAR(J)=CINV(RANUNI(0),DEGREE2);
      P=Z*((A*LMDAD(J))+B)+XX*(LMDAR(J)/(C+LMDAR(J)));
      X(J)=LOG(P);
    END;
    M=MEAN(OF X1-X25);
    S=STD(OF X1-X25);
    DO J=1 TO SMPLSIZE;
      Y=(X(J)-M)/S;
      PNORM=PROBNORM(Y);
      FILE STDFILE;
      PUT @1 I @5 LMDA(J) 10.5 @16 X(J) 15.10
          @32 Y 15.10 @48 PNORM 15.10;
    END;
  END;
RUN;
DATA STDDATA;
  INFILE STDFILE;
  INPUT I 1-3 LMDA 5-14 X 16-30 Y 32-46 PNORM 48-62;
RUN;
PROC SORT DATA=STDDATA;
  BY I Y;
RUN;
DATA MAXIMUM;
  SET STDDATA;
  BY I Y;
  SMPLSIZE=50;
  J+1;
  D=SQRT(SMPLSIZE)*MAX(ABS(PNORM-(J/(SMPLSIZE+1))),
                       ABS(PNORM-((J-1)/(SMPLSIZE+1))));
  IF J=SMPLSIZE THEN J=0;
RUN;
```

```
PROC MEANS DATA=MAXIMUM MAX;
  VAR D;
  BY I;
  OUTPUT OUT=STATS MAX=MA;
RUN;
DATA _NULL_;
  SET STATS;
  FILE SYSOUT MOD;
  PUT @1 '(D,3000,50  ) : D(MAX)= ' @25 MA;
RUN;
*D;
*-------------------------------------------------------------------------;
*  FORMULA (D); DEGREE OF FREEDOM (1) = 3000                              ;
*               DEGREE OF FREEDOM (2) = 3000                              ;
*-------------------------------------------------------------------------;
DATA SAMPLE;
  DEGREE1=3;
  DEGREE2=3;
  ITERATE=100;
  SMPLSIZE=50;
  ARRAY X(50) X1-X50;
  ARRAY LMDAD(50) LMDAD1-LMDAD50;
  ARRAY LMDAR(50) LMDAR1-LMDAR50;
  TI=1;
  TR=6;
  TT=1;
  A=((TI/730)+(TR/8760));
  B=TT/(24*TI);
  C=8.76E3/TR;
  Z=0.5;
  XX=1.0-Z;
  DO I=1 TO ITERATE;
    DO J=1 TO SMPLSIZE;
      LMDAD(J)=CINV(RANUNI(0),DEGREE1);
      LMDAR(J)=CINV(RANUNI(0),DEGREE2);
      P=Z*((A*LMDAD(J))+B)+XX*(LMDAR(J)/(C+LMDAR(J)));
      X(J)=LOG(P);
    END;
    M=MEAN(OF X1-X25);
    S=STD(OF X1-X25);
    DO J=1 TO SMPLSIZE;
      Y=(X(J)-M)/S;
      PNORM=PROBNORM(Y);
      FILE STDFILE;
      PUT @1 I @5 LMDA(J) 10.5 @16 X(J) 15.10
          @32 Y 15.10 @48 PNORM 15.10;
    END;
  END;
RUN;
DATA STDDATA;
  INFILE STDFILE;
```

```
  INPUT I 1-3 LMDA 5-14 X 16-30 Y 32-46 PNORM 48-62;
RUN;
PROC SORT DATA=STDDATA;
  BY I Y;
RUN;
DATA MAXIMUM;
  SET STDDATA;
  BY I Y;
  SMPLSIZE=50;
  J+1;
  D=SQRT(SMPLSIZE)*MAX(ABS(PNORM-(J/(SMPLSIZE+1))),
                       ABS(PNORM-((J-1)/(SMPLSIZE+1))));
  IF J=SMPLSIZE THEN J=0;
RUN;
PROC MEANS DATA=MAXIMUM MAX;
  VAR D;
  BY I;
  OUTPUT OUT=STATS MAX=MA;
RUN;
DATA _NULL_;
  SET STATS;
  FILE SYSOUT MOD;
  PUT @1 '(D,3000,3000) : D(MAX)= ' @25 MA;
RUN;
```